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(Research Article)

Pythagorean triangles with sum of its two legs as Dodecic

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Abstract

Number Theory is almost four thousand years old. In ancient clay tablet in Babylon, integral solutions of Pythagorean equations were listed down. Pythagoras theorem has always fascinated young and old mind alike. More than four hundred proofs of the Pythagoras theorem have been discovered so far. Pythagoras theorem is applied in almost every sphere of science- from geometry to Einstein's theory of relativity. New Pythagorean triangles are being discovered which satisfy certain constraints. Integral solutions of Diophantine equations related to Pythagorean equation are sought by many mathematicians. Applications of these triangles are also being explored in various fields of knowledge. In this paper, we have found nineteen extraordinary Pythagorean Triangles where their sum of two legs are dodecic numbers. These triangles are found by solving the dodecic Diophantine equation using the software *Mathematica.* Some interesting properties of these Pythagorean Triangles are also observed. Their applications can be explored in cryptography.

Keywords: Euclidean formula; Opposite Parity; Primitive Pythagorean Triangle; Dodecic

1. Introduction

The search for special Pythagorean Triangles continues till today. Darbari et al. [1] have discovered special Pythagorean Triangles with sum of their two legs as undecic. Darbari et al. [2] have found methods to use these Pythagorean Triangles in cryptography. Darbari and Darbari [3] have found out special Pythagorean Triangles in connection with Harshad numbers while Darbari and Darbari [4] have found exceptional Pythagorean Triangles with their perimeter as sum of three squares with two sides consecutive.

An attempt has been made to find special Pythagorean Triangles with sum of its two legs as a number which is a twelfth power of a number, i.e., is a dodecic.

2. Definitions

2.1. Dodecic

A number is called a dodecic if it is twelfth power of some number.

2.1.1. Pythagorean Equation

A quadratic equation

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$$X^2 + Y^2 = Z^2$$

is called Pythagorean equation [5] after the famous mathematician and philosopher Pythagoras. It still remains as one of the most important equations of the world in all times.

2.1.2. Pythagorean Triangle [6]

A right angled triangle with sides X, Y and Z is called Pythagorean Triangle if X, Y and Z are positive integers. X and Y are called its legs and Z is called its hypotenuse. Pythagorean triangles satisfy Pythagorean equation $X^2 + Y^2 = Z^2$.

If X, Y and Z satisfy Pythagorean equation and a is positive integer, then aX, aY and aZ also satisfy it. Therefore, one Pythagorean Triangle can generate infinite Pythagorean triangles.

2.1.3. Primitive Pythagorean Triangle

A Pythagorean Triangle is said to be primitive if X, Y and Z are co-primes, i.e., their greatest common divisor is one. Or, in other words, GCD (X, Y, Z) = 1.

2.2. Opposite Parity

Two natural numbers m and n are called of opposite parity if one of them is even and other is odd, i.e., $m \not\cong n \ (mod 2)$.

2.2.1. Euclidean Formula [7]

The positive primitive solutions of Pythagorean Equation with Y even are

$$X = m^2 - n^2, Y = 2mn, Z = \ m^2 + n^2,$$

where m and n are arbitrary integers of opposite parity with $m > n > 0$ and $(m, n) = 1$.

3. Method of Analysis

In the Pythagorean mathematics, primitive solutions of the Pythagorean Equation

$$X^2 + Y^2 = Z^2 \quad (1)$$

is given by 2.6

$$X = m^2 - n^2, Y = 2\,mn, Z = m^2 + n^2 \quad (2)$$

where $m, n \in I$ such that $m > n > 0$ and $(m, n) = 1$ with one of them is odd other even.

3.1. Sum of two legs is a power of twelve, that is, dodecic

If X and Y are two legs of a right angled triangle and Z is the hypotenuse, then

$$X + Y = \beta^{12} \quad (3)$$

$$\Rightarrow m^2 - n^2 + 2\,mn = \beta^{12} \quad (4)$$

Solving this equation using the software Mathematica, the twelve solutions of (4) are given by

$\beta = (m^2 - n^2 + 2\,mn)^{1/12}$, $\beta = -\,(m^2 - n^2 + 2\,mn)^{1/12}$, $\beta = -i\,(m^2 - n^2 + 2\,mn)^{1/12}$,

$\beta = i\,(m^2 - n^2 + 2\,mn)^{1/12}$, $\beta = -\,(-1)^{1/6}\,(m^2 - n^2 + 2\,mn)^{1/12}$, $\beta = (-1)^{1/6}\,(m^2 - n^2 + 2\,mn)^{1/12}$,

$\beta = -\,(-1)^{1/3}\,(m^2 - n^2 + 2\,mn)^{1/12}$, $\beta = (-1)^{1/3}\,(m^2 - n^2 + 2\,mn)^{1/12}$, $\beta = -(-1)^{2/3}(m^2 - n^2 + 2\,mn)^{1/12}$,

$\beta = (-1)^{2/3}\,(m^2 - n^2 + 2\,mn)^{1/12}$, $\beta = -\,(-1)^{5/6}(m^2 - n^2 + 2\,mn)^{1/12}$, $\beta = (-1)^{5/6}\,(m^2 - n^2 + 2\,mn)^{1/12}$.

Solving the integral solutions, using *Mathematica*, using the command

Find Instance [m^2 - n^2 + 2 mn - β^{12}==0&&n<m&&0<m<10^{15}&&0<n<10^{15}&&0< β<10^{11} &&GCD[m,n]==1, {m,n,β}, Integers,10000], we get only 19 solutions. They are as follows:

Table 1 Values of m, n, β

S. N.	m	n	B
1	83221	80030	7
2	17555585	11743968	17
3	144336505	3796728	23
4	707744245	244990478	31
5	4307805349	493243310	41
6	7754194705	5738360402	47
7	13330535741	531116422	49
8	104082680521	31579397642	71
9	120865698101	41398812492	73
10	221522493785	23904378538	79
11	428902751369	81160408968	89
12	622352222549	338094915118	97
13	940927902041	353620702330	103
14	1669585699885	555803980548	113
15	2400863098325	539668450092	119
16	2475813102101	427628706622	119
17	3016237805729	2248102736098	127
18	5469525155081	1455204818410	137
19	8876744496929	4744194947088	151

3.2. Perimeter is a sum of two squares and a dodecic

Table 2 Values of X, Y, Z

S.N.	$X = m^2 - n^2$	$Y = 2mn$	$Z = m^2 + n^2$
1	520933941	13320353260	13330535741
2	170277780307201	412344456922560	446119349077249
3	20818611532109041	1096012899911280	20847441819121009
4	440881582019951541	346781201768598220	560922250641288509
5	18313897962013255701	4249592338352930380	18800475887732767901
6	27198755419808435421	88992727689140142820	93056315626291638629
7	177421098488360336997	14160132892206077404	177985267795796501165
8	9835946029004997671277	6573736711635813462964	11830462739868107991605
9	12894655301694299756137	10007392744795958955384	16322378653189850256265
10	48500795939437368608781	10590715092236784772660	49643634566013358643669
11	177370558148045216549137	69619845417216923754384	190544582115751246199185
12	273014117283032399103477	420828243712405601191564	501630460540327649011325
13	760297715722915224536781	665463171123263720911060	1010392917955640159394581
14	2478598344467483726632921	1855924755724203011673960	3096434474053486851393529

15	5472901580874017122997161	2591340134312259502591800	6055385652923420112614089
16	5946784205807772103963317	2117457509378504521625644	6312516827262181197265085
17	4043724588657578912955837	13561624927563185359010884	14151656412760206577487045
18	27798084358540156025188461	15918558720177147387682420	32033326485587518175044661
19	56289207167783974170751297	84226012777843544666985504	101303978559734596410110785

From equation (2), hypotenuse is $Z = m^2 + n^2$ and from equation (3),

$X + Y = \beta^{12}$

Therefore, perimeter is given by

$X + Y + Z = \beta^{12} + m^2 + n^2$

Substituting the values of m, n from Table 1, we find the values of X, Y and Z from equation (2); $X = m^2 - n^2$, $Y = 2\,mn$,

$Z = m^2 + n^2$

The following table verifies that the perimeter is a sum of two squares and a dodecic,

Table 3 $X + Y + Z = \beta^{12} + m^2 + n^2$

S.N.	**$X + Y + Z = \beta^{12} + m^2 + n^2$**
1	$27171822942 = 7^{12} + 83221^2 + 80030^2$
2	$1028741586307010 = 17^{12} + 17555585^2 + 11743968^2$
3	$42762066251141330 = 23^{12} + 144336505^2 + 3796728^2$
4	$1348585034429838270 = 31^{12} + 707744245^2 + 244990478^2$
5	$41363966188098953982 = 41^{12} + 4307805349^2 + 493243310^2$
6	$209247798735240216870 = 47^{12} + 7754194705^2 + 5738360402^2$
7	$369566499176362915566 = 49^{12} + 13330535741^2 + 531116422^2$
8	$28240145480508919125846 = 71^{12} + 104082680521^2 + 31579397642^2$
9	$39224426699680108967786 = 73^{12} + 120865698101^2 + 41398812492^2$
10	$108735145597687512025110 = 79^{12} + 221522493785^2 + 23904378538^2$
11	$437534985681013386502706 = 89^{12} + 428902751369^2 + 81160408968^2$
12	$1195472821535765649306366 = 97^{12} + 622352222549^2 + 338094915118^2$
13	$2436153804801819104842422 = 103^{12} + 940927902041^2 + 353620702330^2$
14	$7430957574245173589700410 = 113^{12} + 1669585699885^2 + 555803980548^2$
15	$14119627368109696738203050 = 119^{12} + 2400863098325^2 + 539668450092^2$
16	$14376758542448457822854046 = 119^{12} + 2475813102101^2 + 427628706622^2$
17	$31757005928980970849453766 = 127^{12} + 3016237805729^2 + 2248102736098^2$
18	$75749969564304821587915542 = 137^{12} + 5469525155081^2 + 1455204818410^2$
19	$241819198505362115247847586 = 151^{12} + 8876744496929^2 + 4744194947088^2$

4. Results and discussion

We observe that

- The value 119 of β is repeated once.
- Except for 49 and repeated value 119, which are multiple of 7, rest other values of β are prime numbers.
- $2 m^2 + mn = 0 \pmod 2$
- $4 m^2 + 2 mn = 0 \pmod 4$
- $3 m^2 + mn + n^2 = 0 \pmod 3$
- $m + n + \beta = 0 \pmod 4$ or $2 \pmod 4$
- $3 m + 5 n + \beta = 0 \pmod 2$
- $6 m^3 + 3 n^3 + \Phi(\beta) = 0 \pmod 6$ or $4 \pmod 6$
- $m^2 + n^2 + \{\Phi(\beta)\}^2 = 1 \pmod 4$
- $m^3 + n^3 + \{\Phi(\beta)\}^3 = 1 \pmod 4$
- $m^4 + n^4 + \{\Phi(\beta)\}^4 = 1 \pmod 4$
- $X + Y + Z = 0 \pmod 2$
- $2 X + Y + Z = 0 \pmod 3$
- $X + 2 Y + 3 Z = 0 \pmod 4$
- $(Y + Z - X)^2 = 2 (Y + Z) (Z - X)$
- $(X + 2 Y + Z)^2 = (Z - X)^2 + 4 (X + Y) (Y + Z)$
- $X + 2 Y + Z \pm 2 \{(X + Y) (Z - Y)\}^{1/2} = 0 \pmod{16}$ or $= 0 \pmod 4$

5. Conclusion

In conclusion, one may endeavor to discover Pythagorean Triangle which satisfy the conditions similar to that presented in the above problem.

Compliance with ethical standards

Disclosure of conflict of interest

No conflict of interest to disclosed.

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