GSC Advanced Research and Reviews

eISSN: 2582-4597 CODEN (USA): GARRC2

Cross Ref DOI: 10.30574/gscarr

Journal homepage: https://gsconlinepress.com/journals/gscarr/

(Research Article)

Check for updates

2-D exotic soliton in 2-D Heisenberg ferromagnetic chains with Dzyaloshinsky-Moriya interaction

Fabien Kenmogne [1, *], Adoum Kriga [2], Pierre Marie Wakeu Pola [3], Adoum Danao Adile [4] and Martial Nde Ngnihamye [1]

[1] Department of Civil Engineering, Advanced teacher's training college of the technical education, University of Douala, Douala Cameroon.

[2] Department of technology, Faculty of exact and applied science, University of N'djamena, Chad.

[3] Department of Mathematics, Physics and Chemistry, Siantou University Institute of Yaoundé, University of Yaoundé 1 Cameroon.

[4] Department of Industrial Engineering and Maintenance, Polytechnic University of Mongo, Chad.

GSC Advanced Research and Reviews, 2023, 14(01), 095–107

Publication history: Received on 23 November 2022; revised on 10 January 2023; accepted on 13 January 2023

Article DOI: https://doi.org/10.30574/gscarr.2023.14.1.0378

Abstract

The two-dimensional (2D) version of ferromagnet XXZ spin chain with DzyaloshinskyMoriya (DM) interaction, recently introduced by F. Kenmogne and coworkers is reexplored. Firstly by using the Dyson-Maleev transformation, the 2-D discrete nonlinear Schrödinger (DNLS) equation, governing the quantum states behaviors is found. Next using the semidiscrete multiple-scale method, the 2-D DNLS equation is reduced to the 2D extended nonlinear Schrödinger (ENLS) equation which consists of the basic 2-D NLS equation with additional nonlinear dispersive terms. This equation admits the classical 2D pulse quantum states, when additional terms vanish. In addition, this equation admits the 2D compacton and 2D peakon-like boson quantum states. Furthermore, we notice that on the contrary to the classical outcomes where amplitudes of both solutions are free parameters, the amplitudes for two dimensional quantum states are not free parameters since the obtained solutions need to be normalized.

Keywords: 2-D discrete NLS equation; 2-D Linear dispersion; 2-D pulse soliton; 2-D pulse compacton

1. Introduction

More recently, F. Kenmogne and coworkers [1] investigated the possible propagation of transverse compactlike pulse signal propagation in a two-dimensional nonlinear electrical transmission network with the intersite circuit elements (both in the propagation and transverse directions) acting as nonlinear resistances. Pulse compactons being found for the first time by Roseneau and Hyman[2, 3, 4]. Since these pioneering works, a growing number of works had been devoted to this particular discovery, and this in almost all physical domain, and particularly in nonlinear electrical transmission lines [1, 5, 6, 7] and optical fibers[8, 9]. It has been proved that each equation admitting the compacton as a solution for certain range of parameters, could admit other forms of solutions with discontinuous derivatives in the form of peaks or cusps elsewhere. All solutions found in these studies are classical since the output signals obtained are proportional to the input initial conditions.

Nowadays, in the classical domain, intrinsic localized modes (ILMs), well known as discrete breathers or lattice solitons, have attracted enormous attention in many areas of physics [10]. Discrete breathers can be defined as spatially localized and time-periodic excitations that can exist in classical discrete nonlinear systems. Discrete breathers have been

∗ Corresponding author: Fabien Kenmogne

extensively studied in theoretical and computational settings [11], in order to understand the physical mechanism of energy transfer in low dimensional system.

Recently, some works have been devoted on the studies of quantum soliton like breathers in Heisenberg spin chains, namely by using numerical diagonalization, and nondegenerate and degenerate perturbation theory, Djoufack et al [12, 13] calculating energy spectrum of anisotropic ferromagnetic Heisenberg spin chains, shown that two, four, and six-quanta quantum breathers can exist in these ferromagnetic chains. While recently, Bing Tang et al [14] have analytically shown that quantum breathers with a large number of quanta are possible in ferromagnetic chains with on-site easy axis anisotropy. More recently, Djoufack et al [15] have constructed the quantum soliton and theirs properties in 1D Heisenberg spin chains included Dzyaloshinsky-Moriya interaction for a long range interactions.

In this paper, we reconsider the two-dimensional (2D) version of the Heisenberg ferromagnetic XXZ spin chain with DM interaction recently introduced by F. Kenmogne et al. [16]. Let us remember that the DM interaction was proposed first by Dzyaloshinsky and Moriya to describe weak ferromagnet, which is essentially an antisymmetric spin coupling that appears when the symmetry around the magnetic ions is not high enough, thus leading to the mechanism of weak ferromagnetism, which is caused by the combined effect of spin-orbit coupling and spin-spin exchange interaction [17, 18]. In magnetic systems, weak ferromagnetism plays an important role in depicting insulators, quantum phase transitions, spin soliton excitations [19]. one may wonder whether the weak ferromagnetic systems may admit 2D-compacton and 2-D peakon as quantum excitations, analogous to their classical counterparts. The answer to this question is the main objective of the present work.

Thus the paper is structured as follows: In Sec.2, we present the model description and the derivation of the 2-D DNLS equation. In Sec.3, we use the semi-discrete multiple-scale method to derive the 2-D ENLS equation governing weak amplitude modulated waves. In Sec.4, we obtain the stationary localized exotic solitons as solution namely: the bright compacton and peakon-like quantum breathers. Finally, in Sec.5, we give some concluding remarks.

2. Model description and derivation of the discrete NLS equation

2.1. Hamiltonian description and its Bosonization

The starting point here is the Hamiltonian for an anisotropic ferromagnetic chain with uniform DM interaction, given by[14]:

$$H = -\sum_{i}^{f}\sum_{j}^{f}\left[J_1\left(S_{ij}^{x}S_{\{i+1,j\}}^{x} + S_{\{ij\}}^{x}S_{\{i,j+1\}}^{x} + S_{\{ij\}}^{y}S_{\{i+1,j\}}^{y} + S_{\{ij\}}^{y}S_{\{i,j+1\}}^{y}\right) + J_2\left(S_{\{ij\}}^{z}S_{\{i+1,j\}}^{z} + S_{\{ij\}}^{z}S_{\{i,j+1\}}^{z}\right)\right]$$

$$+\vec{D}\sum_{\{i\}}^{f}\sum_{\{j\}}^{f}\left(\vec{S}_{\{ij\}}\times\vec{S}_{\{i+1,j\}} + \vec{S}_{\{ij\}}\times\vec{S}_{\{i,j+1\}}\right), \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

with $\vec{S}_{ij} = \left(S_{\{ij\}}^{x};\ S_{\{ij\}}^{y}\ ;\ S_{\{ij\}}^{z}\right)$, where $S_{\{ij\}}^{m}$ $(m = x;\ y;\ z)$ is the $m^{\{th\}}$ component of the spin operator on the site j, J_1 and J_2 are the exchange constants, $\vec{D} = D\overrightarrow{ez}$, D being the DM interaction parameter, while f is the number of sites in this magnetic lattice. By setting $S^{\{(ij)\pm\}} = S_{\{ij\}}^{x} \pm I\, S_{\{ij\}}^{y}$, with I^2 $= -1$, the Hamiltonian (1) can be rewritten as

$$H = -\frac{1}{2}\sum_{\{i\}}^{f}\sum_{\{j\}}^{f}[J_1(S_{\{ij\}}^{+}S_{\{i+1,j\}}^{-} + S_{\{ij\}}^{+}S_{\{i,j+1\}}^{-} + S_{\{ij\}}^{-}S_{\{i+1,j\}}^{+} + S_{\{ij\}}^{-}S_{\{i,j+1\}}^{+}) + 2J_2S_{\{ij\}}^{z}S_{\{i+1,j\}}^{z} + 2J_2S_{\{ij\}}^{z}S_{\{i,j+1\}}^{z}] - I\,\frac{D}{2}\sum_{i}^{f}\sum_{j}^{f}(S_{\{ij\}}^{-}S_{\{i+1,j\}}^{+} + S_{\{ij\}}^{-}S_{\{i,j+1\}}^{+} - S_{\{ij\}}^{+}S_{\{i+1,j\}}^{-} - S_{\{ij\}}^{+}S_{\{i,j+1\}}^{-}), \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

where $S_{\{ij\}}^{+}, S_{\{ij\}}^{-}$ and S_{ij}^{z} are spin operators satisfying the commutation relations $[S_{\{ij\}}^{+}, S\_\{i'j'\}] = 2S_{\{ij\}}^{z}\delta_{\{ij\}}\delta_{\{i'j'\}}$

and $\left[S_{\{ij\}}^{\pm}, S_{\{i'j'\}}^{z}\right] = \mp S_{\{ij\}}^{\pm}\delta_{\{ij\}}\delta_{\{i'j'\}}$, with $\overrightarrow{S_{\{ij\}}}.\overrightarrow{S_{\{ij\}}} = S(S+1)$. For sake of simplicity, the Planck constant is set to $\hbar = 1$ in this paper. To obtain the new version of (2) as function of creation and annihilation operators, let us introduce the Dyson-Maleev transformation as [20]:

$$S_{\{ij\}}^{+} = (2S)^{\frac{1}{2}}\left(1 - \frac{a_{ij}^{\dagger}a_{\{ij\}}}{2S}\right)a_{\{ij\}}, S_{\{ij\}}^{-} = (2S)^{\frac{1}{2}}a_{ij}^{\dagger} \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$S^z_{\{ij\}} = S - a^\dagger_{ij} a_{\{ij\}}. \quad \text{.................. (4)}$$

where $a_{\{ij\}}(a^\dagger_{ij})$ is a boson annihilation (creation) operator, leading by setting $\alpha = J_1 + ID$ in (1) to the following bosonized Hamiltonian:

$$H = -\sum_{\{i\}}^{f}\sum_{\{j\}}^{f} S\left[\alpha^*\, a^\dagger_{\{i+1,j\}}a_{\{ij\}} + \alpha^*\, a^\dagger_{\{i,j+1\}}a_{\{ij\}} + \alpha\, a^\dagger_{\{ij\}}a_{\{i+1,j\}} + \alpha\, a^\dagger_{\{ij\}}a_{\{i,j+1\}} - J_2\left(a^\dagger_{\{i+1,j+1\}}\, a_{\{i+1,j+1\}} + a^\dagger_{\{ij\}}\, a_{\{ij\}}\right)\right] + \frac{1}{2}\sum_{\{i\}}^{f}\sum_{\{j\}}^{f}(\alpha\, a^\dagger_{\{i+1,j\}}a^\dagger_{\{ij\}}\, a_{\{i+1,j\}}\, a_{\{i+1,j\}} + \alpha\, a^\dagger_{\{i,j+1\}}a^\dagger_{\{ij\}}\, a_{\{i,j+1\}}\, a_{\{i,j+1\}} + \alpha^* a^\dagger_{\{i+1,j\}}a^\dagger_{\{ij\}}\, a_{\{ij\}}a_{\{ij\}} + \alpha^* a^\dagger_{\{i,j+1\}}a^\dagger_{\{ij\}}\, a_{\{ij\}}a_{\{ij\}} - 2J_2(\, a^\dagger_{\{i+1,j\}}a^\dagger_{\{ij\}}a_{\{i+1,j\}}a_{\{ij\}} + a^\dagger_{\{i,j+1\}}a^\dagger_{\{ij\}}a_{\{i,j+1\}}a_{\{ij\}})\,)\,. \quad \text{.................. (5)}$$

To avoid overloading the paper, the ground state energy is neglected.

2.2. Quantum dynamics Analysis and time-dependent Hartree approximation

In the quantum mechanics point of view, one can use these following three different methods which seem to be equivalent and that are the Dirac interaction, the Schrödinger and Heisenberg picture. In this present work, the Schrödinger picture is adopted to analyze the quantum dynamic of our system. Thus, the state vector $|\Psi(t)\rangle$ is time dependent, while operators are time independent. The time evolution of the state vector $|\Psi(t)\rangle$ of the system is governed by the Schrödinger equation

$$I\,\frac{d|\Psi(t)\rangle}{dt} = H|\Psi(t)\rangle. \quad \text{.................. (6)}$$

The Hamiltonian (H) in Eq. (6) commutes with the number operator defined as $\widehat{N} = \sum_{\{i\}}^{f}\sum_{\{j\}}^{f} a^\dagger_{\{ij\}}\, a_{\{ij\}}$ whose eigenvalue is n. Thus, the boson number is conserved. Taking in consideration H in (5) and $\widehat{N}$, we can deduce that the boson number is conserved and obviously, one can rewrite the system using the Fock representation, in which the general n-boson system state vector is expanded as [14]

$$|\Psi(t)\rangle = \frac{1}{\sqrt{n!}}\sum_{\{i_1,j_1=1\}}^{f}\sum_{\{i_2,j_2=1\}}^{f}\cdots\sum_{\{i_n,j_n=1\}}^{f}\theta_n\,(i_1,i_2,\ldots,i_n,j_1,j_2,\ldots,j_n,t)\; a^\dagger_{\{i_1j_1\}}\, a^\dagger_{\{i_2j_2\}}\cdots a^\dagger_{\{i_nj_n\}}|0\rangle \quad \text{.................. (7)}$$

where $|0\rangle \equiv |0\rangle 1|0\rangle 2 \cdots |0\rangle f$ is the vacuum state. θn are fn time dependent coefficients of corresponding number states, while more generally θn ($i1, i2, ..., in, j1, j2, ..., jn, t$) is the n-boson wave function, which need to be normalized as

$$\sum_{\{i_1,j_1=1\}}^{f}\sum_{\{i_2,j_2=1\}}^{f}\cdots\sum_{\{i_n,j_n=1\}}^{f}|\theta_n(i_1,i_2,\ldots,i_n,j_1,j_2,\ldots,j_n,t)\,|^2 = 1. \quad \text{.................. (8)}$$

Taking into account the bosonized Hamiltonian (5) and the state vector (7) into the Schrödinger equation (6) and considering the boson commutation relations $[a_{\{ij\}}, a^\dagger_{\{ij\}}] = \delta_{\{ij\}}\delta_{\{i'j'\}}$, the following Schrödinger equation for the n-boson wave function:

$$\left(I\frac{d}{dt} - n\omega_0\right)\theta_n(i_1,i_2,\ldots,i_n,j_1,j_2,\ldots,j_n,t) = -S\sum_{\{k=1\}}^{n}\sum_{\{l=1\}}^{n}\Big[\alpha^*\theta_n\left(i_1,i_2,\ldots,i_{\{k-1\}},i_{\{k\}}-1,i_{\{k+1\}},\ldots,i_n,j_1,j_2,\ldots,j_{\{l-1\}},j_{\{l\}}-1,j_{\{l+1\}},\ldots,j_n,t\right) + \alpha\theta_n(i_1,i_2,\ldots,i_{\{k-1\}},i_{\{k\}}+1,i_{\{k+1\}},\ldots,i_n,j_1,j_2,\ldots,j_{\{l-1\}},j_{\{l\}}+1,j_{\{l+1\}},\ldots,j_n,t\,)\Big] + \sum_{\{k=1\}}^{n}\sum_{\{l\neq k\}}^{n}\Big[\frac{\alpha}{2}\delta_{\{ikik+1jljl+1\}}\theta_n\big(i_1,i_2,\ldots,i_l,\ldots,i_{\{k-1\}},i_{\{k\}}+1,i_{\{k+1\}},\ldots,i_n,j_1,j_2,\ldots,j_l,\ldots,j_{\{l-1\}},j_{\{l\}}+1,j_{\{l+1\}},\ldots,j_n,t\big) + \frac{\alpha^*}{2}\delta_{\{ikik-1jljl-1\}}\theta_n(i_1,i_2,\ldots,i_l,\ldots,i_{\{k-1\}},i_{\{k\}}-1,i_{\{k+1\}},\ldots,i_n,j_1,j_2,\ldots,j_l,\ldots,j_{\{l-1\}},j_{\{l\}}-1,j_{\{l+1\}},\ldots,j_n,t\,) - J_2\delta_{\{ikik+1jljl+1\}}\theta_n\left(i_1,i_2,\ldots,i_l,\ldots,i_{\{k\}},\ldots,i_n,j_1,j_2,\ldots,j_l,\ldots,j_{\{l\}},\ldots,j_n,t\right)\Big], \quad \text{.................. (9)}$$

is obtained. Where $\omega_0 = 2SJ_2$, and where the interaction between pairs of bosons is a Kronecker delta-function. This can be compared with the corresponding quantum field theory for a Bose gas involving a Dirac delta function interaction. Equation (9) is the set of ordinary differential equation, difficult to solve exactly, this is why it is natural to turn to approximate methods [21, 22, 23]. In our work, we use the time-dependent Hartree approximation, usually applied to the studies of nonlinear excitations in optical fibers and lattice systems and which is well known in quantum field theory [21, 23]. This approximation is used when the number of boson becomes large and the exact eigenfunctions of the Hamiltonian very difficult to construct. Its basic idea is the fact that a system of n-bosons can be described by a single-particle wave function, due to the fact that every boson feels the same potential caused by the interaction with other bosons. In this approximation, therefore, n-boson wave function θ(i1, i2, ..., in, j1, j2, ..., jn, t) is assumed to be rewritten as a product of the form [24].

$$\theta_n^H(i_1, i_2, \ldots, i_n, j_1, j_2, \ldots, j_n, t) = \prod_{\{kl\}}^{\{n\}} \Phi_{\{n,kl\}}(t) \ldots \ldots \ldots \ldots \ldots \ldots (10)$$

where Φn,kl is the single-boson wave function with *ik* = 1, 2, ..., *f*, *jl* = 1, 2, ..., *f*, *l* = 1, 2, ..., *n* and *k* = 1, 2, ..., *n* labeling the boson. Accounting to Eq. (10), the n-boson state vector (7) can be reduced to

$$|\Psi_n(t)\rangle^H = \frac{1}{\sqrt{n!}}\left(\sum_{\{i=1\}}^{f}\sum_{\{j=1\}}^{f}\Phi_{\{n,ij\}}(t)\, a_{\{ij\}}^{\dagger}\right)^n |0\rangle. \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

and from Eq. (8) the normalization then is $\rho = 1$, where

$$\rho = \sum_{\{i=1\}}^{f}\sum_{\{j=1\}}^{f}\left|\Phi_{\{n,ij\}}(t)\right|^2 \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

is the norm. The functions $\Phi_{nij}(t)$ are to be determined by extremizing the action integral $S^H = \int dt\, (Ln(t))$ [25]. Where $Ln(t)$ given by

$$L_{\{n\}}(t) = nI\sum_{\{ij\}}^{f}[I\Phi_{\{n,ij\}}^*\frac{d\Phi_{\{n,ij\}}}{dt} - \omega_0\Phi_{\{n,ij\}}^*\Phi_{\{n,ij\}} + S\,\alpha^*\Phi_{\{n,ij\}}^*\Phi_{\{n,i-1,j\}} + S\alpha^*\Phi_{\{n,ij\}}^*\Phi_{\{n,i,j-1\}} + S\alpha\Phi_{\{n,ij\}}^*\Phi_{\{n,i+1,j\}} + S\alpha\Phi_{\{n,ij\}}^*\Phi_{\{n,i,j+1\}} - \frac{\alpha}{2}(n-1)[\Phi_{\{n,j\}}^*\Phi_{\{n,i-1,j\}}^*\Phi_{\{n,ij\}}\Phi_{\{n,ij\}} + \Phi_{\{n,ij\}}^*\Phi_{\{n,i,j-1\}}^*\Phi_{\{n,ij\}}\Phi_{\{n,ij\}}] - \frac{\alpha^*}{2}(n-1)[\Phi_{\{n,ij\}}^*\Phi_{\{n,i+1,j\}}^*\backslash Phi_{\{n,ij\}}\backslash Phi_{\{n,ij\}} + \Phi_{\{n,ij\}}^*\Phi_{\{n,i,j+1\}}^*\Phi_{\{n,ij\}}\Phi_{\{n,ij\}}] + J_2(n-1)[\Phi_{\{n,ij\}}^*\Phi_{\{n,i-1,j\}}^*\,\Phi_{\{n,ij\}}\Phi_{\{n,i-1,j\}} + \Phi_{\{n,ij\}}^*\Phi_{\{n,i,j-1\}}^*\,\Phi_{\{n,ij\}}\Phi_{\{n,i,j-1\}}]], \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

is the Lagrangian. Requiring $\delta\, S^H/\delta\Phi_{\{n,ij\}}^* = 0$for the optimal Hartree solution, the above action integral leads to the following discrete NLS equation of motion for the single-boson wave function:

$$I\frac{d\Phi_{\{ij\}}}{dt} - \omega_0\Phi_{\{ij\}} + S\left[\alpha^*\left(\Phi_{\{i-1,j\}} + \Phi_{\{i,j-1\}}\right) + \alpha\left(\Phi_{\{i+1,j\}} + \Phi_{\{j,j+1\}}\right)\right] + (n-1)\left[-\frac{\alpha}{2}\left(\Phi_{\{i-1,j\}}^*\Phi_{\{ij\}}^2 + \Phi_{\{i,j-1\}}^*\Phi_{\{ij\}}^2 + \left|\Phi_{\{i+1,j+1\}}\right|^2\Phi_{\{i+1,j+1\}}\right) - \frac{\alpha^*}{2}\left(\Phi_{\{i+1,j\}}^*\Phi_{\{ij\}}^2 + \Phi_{\{i,j+1\}}^*\Phi_{\{ij\}}^2 + \left|\Phi_{\{i-1,j-1\}}\right|^2\Phi_{\{i-1,j-1\}}\right) + J_2\left(\left|\Phi_{\{i-1,j\}}\right|^2 + \left|\backslash Phi_{\{i,j-1\}}\right|^2 + \left|\Phi_{\{i+1,j\}}\right|^2 + \left|\Phi_{\{i,j+1\}}\right|^2\right)\Phi_{\{ij\}}\right] = 0. \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

We now proceed to derive the Hamiltonian for the discrete Eq. (14) defined as :

$$H_{\{n\}}(t) = \partial\{\frac{L_{\{n\}}(t)}{\partial\left(\partial\Phi_{\{n,ij\}}/\partial t\right)}\frac{\partial\Phi_{\{n,ij\}}}{\partial t} - L_{\{n\}}(t) \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

and whose the expression is explicitly given by

$$H_{\{n\}}(t) = -nI\sum_{\{ij\}}^{f} -\omega_0\Phi_{\{n,ij\}}^*\Phi_{\{n,ij\}} + S\,\alpha^*\Phi_{\{n,ij\}}^*\Phi_{\{n,i-1,j\}} + S\alpha^*\Phi_{\{n,ij\}}^*\Phi_{\{n,i,j-1\}} + S\alpha\Phi_{\{n,ij\}}^*\Phi_{\{n,i+1,j\}} + S\alpha\Phi_{\{n,ij\}}^*\Phi_{\{n,i,j+1\}} - \frac{\alpha}{2}(n-1)[\Phi_{\{n,j\}}^*\Phi_{\{n,i-1,j\}}^*\Phi_{\{n,ij\}}\Phi_{\{n,ij\}} + \Phi_{\{n,ij\}}^*\Phi_{\{n,i,j-1\}}^*\Phi_{\{n,ij\}}\Phi_{\{n,ij\}}] - \frac{\alpha^*}{2}(n-1)[\Phi_{\{n,ij\}}^*\Phi_{\{n,i+1,j\}}^*\backslash Phi_{\{n,ij\}}\backslash Phi_{\{n,ij\}} + \Phi_{\{n,ij\}}^*\Phi_{\{n,i,j+1\}}^*\Phi_{\{n,ij\}}\Phi_{\{n,ij\}}] + J_2(n-1)[\Phi_{\{n,ij\}}^*\Phi_{\{n,i-1,j\}}^*\,\Phi_{\{n,ij\}}\Phi_{\{n,i-1,j\}} + \Phi_{\{n,ij\}}^*\Phi_{\{n,i,j-1\}}^*\,\Phi_{\{n,ij\}}\Phi_{\{n,i,j-1\}}]], \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

which is identified as the total 'energy' of the system in a real application.

3. Reduction of the equation of motion: 2D-Extended NLS equation

In this section, we use the semi-discrete multiple-scale method [26] to solve Eq.(14) approximately, by first seeking modulated wave solutions of the form:

$$\Phi_{\{ij\}}(t) = \psi_{\{ij\}}(t)exp\left(\theta_{\{ij\}}(t)\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

where $\theta_{ij}(t) = k_1 ia + k_2 ja - \omega t$ stands for the phase of the carrier wave, which leads Eq. (14) to

$$I\frac{d\Psi_{\{ij\}}}{dt} + (\omega - \omega_0)\Psi_{\{ij\}} + S\sqrt{J_1^2 + D^2}[(\Psi_{\{i-1,j\}} + \Psi_{\{i+1,j\}})cos(a(k_0 + k_1)) + (\Psi_{\{i,j-1\}} + \Psi_{\{i,j+1\}})cos(a(k_0 + k_2)) + I(\Psi_{\{i+1,j\}} - \Psi_{\{i-1,j\}})sin(a(k_0 + k_1)) + I(\Psi_{\{i,j+1\}} - \Psi_{\{i,j-1\}})\, sin(a(k_0 + k_2))] - \frac{(n-1)\sqrt{J_1^2 + D^2}}{2}[\Psi_{\{ij\}}^2[(\Psi_{\{i-1,j\}}^* + \Psi_{\{i+1,j\}}^*)cos(a(k_0 + k_1)) + (\Psi_{\{i,j-1\}}^* + \Psi_{\{i,j+1\}}^*)cos(a(k_0 + k_2)) + I(\Psi_{\{i-1,j\}}^* - \Psi_{\{i+1,j\}}^*)sin(a(k_0 + k_1)) + I(\Psi_{\{i,j-1\}}^* - \Psi_{\{i,j+1\}}^*)sin(a(k_0 + k_2))] + \left(\left|\Psi_{\{i+1,j+1\}}\right|^2\Psi_{\{i+1,j+1\}} + \left|\Psi_{\{i-1,j-1\}}\right|^2\Psi_{\{i-1,j-1\}}\right) cos(a(k_0 + k_1 + k_2))) + I\left(\left|\Psi_{\{i+1,j+1\}}\right|^2\Psi_{\{i+1,j+1\}} - \left|\Psi_{\{i-1,j-1\}}\right|^2\Psi_{\{i-1,j-1\}}\right) sin(a(k_0 + k_1 + k_2))))] + J_2(n-1)\left(\left|\Psi_{\{i-1,j\}}\right|^2 + \left|\Psi_{\{i,j-1\}}\right|^2 + \left|\Psi_{\{i+1,j\}}\right|^2 + \left|\Psi_{\{i,j+1\}}\right|^2\right)\Psi_{\{ij\}} = 0. \quad \text{.................. (18)}$$

with $k_0 = \frac{1}{a}\, arctan\left(\frac{D}{J_1}\right)$. The basic of semi-discrete approximation is assumption that envelope function Ψ is regarded as continuum variable. In this approximation, therefore, we need to adopt the continuum approximation for $\Psi ij(t) \equiv \Psi(x,\ y,\ \tau)$, with $\tau = t$, and $x = ia - v_{g1}t, y = ja - v_{g2}t$. Next, we power expand $\Psi ij\pm 1$ to second order as:

$$\Psi_{\{i\pm 1j\}} = \Psi \pm \frac{\partial\Psi}{\partial x} + \frac{a^2}{2}\frac{\partial^2\Psi}{\partial x^2},\ \Psi_{\{ij\pm 1\}} = \Psi \pm \frac{\partial\Psi}{\partial y} + \frac{a^2}{2}\frac{\partial^2\Psi}{\partial y^2} \ldots\ldots\ldots\ldots\ldots\text{(19)}$$

which leads Eq.(18) to:

$$I\left(\frac{\partial\Psi}{\partial\tau} - v_{\{g1\}}\frac{\partial\Psi}{\partial x} - v_{\{g2\}}\frac{\partial\Psi}{\partial y}\right) + \left[\omega - \omega_0 + 2S\sqrt{J_1^2 + D^2}\left(cos(a(k_0 + k_1)) + \backslash cos(a(k_0 + k_2))\right)\right]\Psi$$

$$S\sqrt{J_1^2 + D^2}\left[a^2\frac{\partial^2\Psi}{\partial x^2}cos(a(k_0 + k_1)) + a^2\frac{\partial^2\Psi}{\partial y^2}cos(a(k_0 + k_2)) + 2Ia\frac{\partial\Psi}{\partial x}sin(a(k_0 + k_1)) + 2Ia\frac{\partial\Psi}{\partial y}sin(a(k_0 + k_2))\right] - \frac{(n-1)a^2\sqrt{J_1^2 + D^2}}{2}\left[\Psi^2\frac{\partial^2\Psi^*}{\partial x^2}cos(a(k_0 + k_1)) + \frac{\partial^2\Psi^*}{\partial y^2}cos(a(k_0 + k_2)) + \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)^2(|\Psi|^2\Psi)cos(a(k_0 + k_1 + k_2))\right] + J_2(n-1)\left(\frac{\partial^2|\Psi|^2}{\partial x^2} + \frac{\partial^2|\Psi|^2}{\partial y^2}\right)\Psi -$$

$$(n-1)\left[\sqrt{J_1^2 + D^2}\left(cos(a(k_0 + k_1)) + cos(a(k_0 + k_2)) + cos(a(k_0 + k_1 + k_2))\right) - 4J_2\right]|\Psi|^2\Psi + Ia(n-1)\sqrt{J_1^2 + D^2}\left[\left(\frac{\partial|\Psi|^2}{\partial x} + \frac{\partial|\Psi|^2}{\partial y}\right)sin(a(k_0 + k_1 + k_2)) - \Psi^2\left(\frac{\partial\Psi^*}{\partial x}sin(a(k_0 + k_1)) + \frac{\partial\Psi^*}{\partial y}sin(a(k_0 + k_2))\right)\right] = 0. \quad \text{.................. (20)}$$

For the weak amplitude quantum breathers with constant shape, Eq. (20) leads to the dispersion relation:

$$\omega = \omega_0 - 2S\sqrt{\{J_1^2 + D^2\}}\left(cos((k_1 + k_0)a) + cos((k_2 + k_0)a)\right) \text{.................. (21)}$$

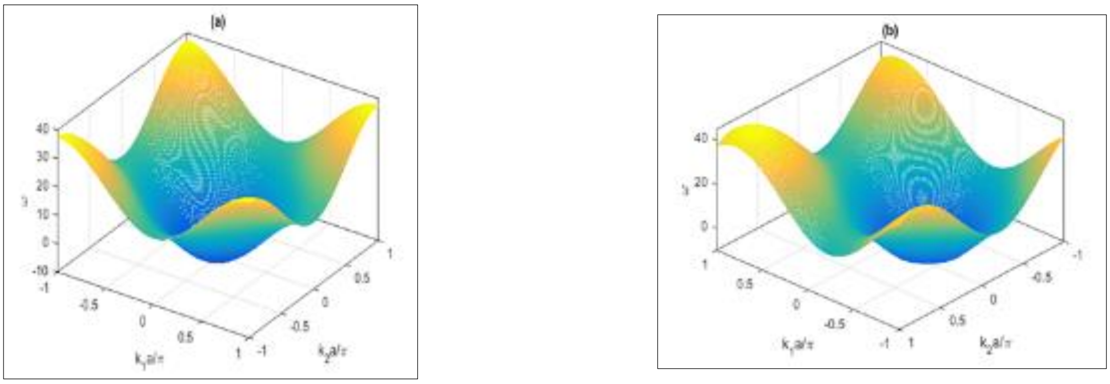

Figure 1 The dispersion curves of linear spin waves for different values of D=J1. In all cases, we fix $J_1 = 0.4$ and $J_2 = 0.45$. (a) $D/J_1 = 0$, (b) $D/J_1 = 0.8$

This dispersion relation, plotted in Fig(1) is the band pass filter, with the maximum $\omega_{\max} = \omega_0 + 4S\sqrt{\{J_1^2 + D^2\}}$, obtained at $k1 = k2 = \pi/a\text{-}k0$, and the minimum $\omega_{\min} = \omega_0 - 4S\sqrt{\{J_1^2 + D^2\}}$, obtained at $k1 = k2 = \text{-}k0$. From the dispersion relation (21), the coordinate of group velocity in x and y directions can easily been found as:

$$v_{\{g1\}} = \frac{d\omega}{dk_1} = 2Sa\sqrt{\{J_1^2 + D^2\}}sin\big((k_1 + k_0)a\big), v_{\{g2\}} = \frac{d\omega}{dk_2} = 2Sa\sqrt{\{J_1^2 + D^2\}}sin\big((k_2 + k_0)a\big) \text{ .... (22)}$$

Taking into account the dispersion relation (21) and group velocity coordinates (22) into Eq.(20), one has:

$$I\frac{\partial\Psi}{\partial\tau} + P_x\frac{\partial^2\Psi}{\partial x^2} + P_y\frac{\partial^2\Psi}{\partial y^2} + Q|\Psi|^2\Psi + \Psi^2\left(\gamma_{\{1x\}}\frac{\partial^2\Psi^*}{\partial x^2} + \gamma_{\{1y\}}\frac{\partial^2\Psi^*}{\partial y^2}\right) + \gamma_2\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)^2(|\Psi|^2\Psi) + \gamma_3\left(\frac{\partial^2|\Psi|^2}{\partial x^2} + \frac{\partial^2|\Psi|^2}{\partial y^2}\right)\Psi + \mathrm{I}\left[\left(\frac{\partial(|\Psi|^2\Psi)}{\partial x} + \frac{\partial(|\Psi|^2\Psi)}{\partial y}\right)\chi_1 - \Psi^2\left(\chi_x\frac{\partial\Psi^*}{\partial x} + \chi_y\frac{\partial\Psi^*}{\partial y}\right)\right] = 0 \text{ .... (23)}$$

With

$$P_x = Sa^2\sqrt{J_1^2 + D^2}cos\big((k_1 + k_0)a\big), P_y = Sa^2S\sqrt{J_1^2 + D^2}cos\big((k_2 + k_0)a\big), Q = (n-1)\Big(4J_2 - S\sqrt{J_1^2 + D^2}\Big(cos\big(a(k_0 + k_1)\big) + < cos\big(a(k_0 + k_2)\big) + cos\big(a(k_0 + k_1 + k_2)\big)\Big)\Big), \chi_x = \frac{n-1}{2S}v_{\{g1\}}, \chi_y = \frac{n-1}{2S}v_{\{g2\}}, \chi_1 = a(n-1)\sqrt{J_1^2 + D^2}sin\big(a(k_0 + k_1 + k_2)\big), \gamma_{\{1x\}} = -\frac{(n-1)}{2S}P_x, \gamma_{\{1y\}} = -\frac{(n-1)}{2S}P_y, \gamma_2 = -\frac{(n-1)}{2}a^2\sqrt{J_1^2 + D^2}cos\big(a(k_0 + k_1 + k_2)\big), \gamma_3 = J_2(n-1) \text{ .... (24)}$$

4. Solution of the Extended NLS equation

4.1. Preliminary: 2D ordinary solitons

For weak value of a, that is for $a \equiv 0$, Eq.(23) reduces to

$$I\frac{\partial\Psi}{\partial\tau} + P_x\frac{\partial^2\Psi}{\partial x^2} + P_y\frac{\partial^2\Psi}{\partial y^2} + Q|\Psi|^2\Psi = 0. \text{ .... (25)}$$

admitting as solution the 2D transverse pulse soliton:

$$\Psi(x,y,\tau) = \Psi_0 sech(\mu_1 x + \mu_2 y - v_e\tau)exp\Big(I\big(\eta_1 x + \eta_2 y - v_p\tau\big)\Big) \text{ .... (26)}$$

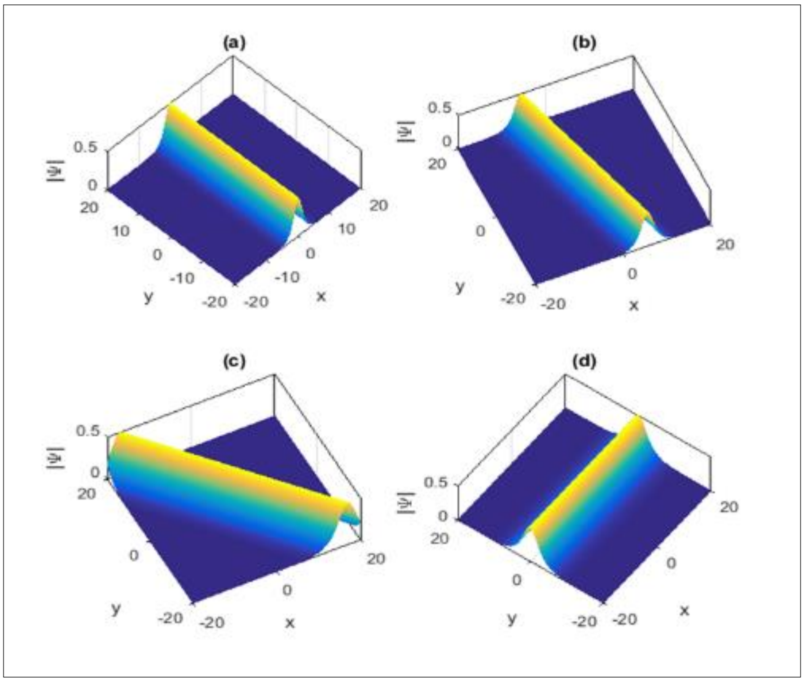

Figure 2 Profile of 2D pulse soliton given by Eq.(26). with parameters $P_x = 0.5$, $P_y = 0.2$, Q = 1 and $\Psi_0 = 0.5$, (a): μ1 = 0, (b): $\mu_1 = 0.2$. (c) $\mu_1 = 0.4$, (d) $\mu_1 = 0.5$, while μ_2 is deduced from Eq.(28)

where v_e and v_p are the envelope and phase velocities, with

$$v_e = 2P_y\mu_2\eta_2 + 2P_x\mu_1\eta_1, v_p = P_x(\eta_1^2 - \mu_1\text{^}2) + P_y(\eta_2^2 - \mu_2^2) \text{.................} (27)$$

while μ_1 and μ_2 are related to soliton widths, obeying the constraints $P_x\mu_1^2 + P_y\mu_2^2 = \frac{Q}{2}\Psi_0^2$. By setting $\mu_1 = \mu cos(\vartheta)$, $\mu_2 = \mu sin(\vartheta)$, one has

$$\mu = \Psi_0\sqrt{\frac{Q}{2\left(P_x cos^2(\vartheta) + P_y sin^2(\vartheta)\right)}}. \text{.................} (28)$$

The profile of this solution is given by Fig.2. It is obvious that this equation has a solution if PxQ > 0 and/or PyQ > 0 since ϑ is a free variable. In Fig.(5), one has in yellow color the domain where PxQ > 0 and PyQ > 0, and then 2-D pulse soliton can exist ∀ϑ. Otherwise the green color is obtained for PxPyQ < 0, and 2-D pulse soliton can exist only for some values of ϑ. By taking into account the normalization condition in the transverse direction z = aμ(i cos(ϑ) + j cos(ϑ)), it is obvious that:

$$\rho = \sum_{\{i=1\}}^{\{f_1\}}\sum_{\{j=1\}}^{\{f_2\}} |\Phi_{\{n,i,j\}}(t)|\text{^}2 = \frac{\Psi_0^2}{a\mu}\int_{\{-\infty\}}^{\{\infty\}} sech^2(z)dz = \frac{2\Psi_0^2}{a\mu} = 1, \text{.................} (29)$$

leading by taking into account to Eq.(28) to the soliton amplitude

$$\Psi_0 = \frac{a}{2}\sqrt{\frac{Q}{2\left(P_x cos^2(\vartheta) + P_y sin^2(\vartheta)\right)}}. \text{.................} (30)$$

Remembering to original variables, it is obvious that!

$$\Phi_{\{ij\}}(t) = \Psi_0 sech[a\mu\left(icos(\vartheta) + jsin(\vartheta)\right) - (ve + \mu(v_{\{g1\}}cos(\vartheta) + v_{\{g2\}}sin(\vartheta)))t]exp\, I\left[(k_1 + \eta_1)ia + (k_2 + \eta_2)ja - \left(\omega + \eta_1 v_{\{g1\}} + \eta_2 v_{\{g2\}} + v_p\right)t\right] \ldots\ldots\ldots\ldots\ldots\ldots (31)$$

For the stationary breather, one has $v_e + \mu(v_{g1}\, cos(\vartheta) + v_{g2}\, sin(\vartheta)) = 0$. By substituting Eq. (31) into Eq. (11) and using Eq.(30), one can construct the following Hartree product eigenstates:

$$|\Psi_n(t)\rangle^{\{(H)\}} = \frac{1}{\sqrt{n!}} exp\big(-nI(\omega + \eta_1\, v_{\{g1\}} + \eta_2\, v_{\{g2\}} + v_p)t\big)\Psi_0^n \left(\sum_{\{i=1\}}^{f}\sum_{\{j=1\}}^{f}(sech[a\mu(icos(\vartheta) + jsin(\vartheta))]\, exp\, I[(k_1 + \eta_1)ia + (k_2 + \eta_2)ja]a_{\{ij\}}^{\dagger}\right)^n |0\rangle. \ldots\ldots\ldots\ldots\ldots\ldots (32)$$

We can then obtain the mean number of bosons on $z_{\{ij\}} = icos(\vartheta) + jsin(\vartheta)$ direction, which has the following form $\langle n_{\{ij\}}(t)\rangle^{\{(H)\}} = \langle \Psi_n(t)|a_{\{ij\}}^{+}\, a_{\{ij\}}|\Psi_n(t)\rangle\, |\Psi_n(t)\rangle^{\{(H)\}}$, that is:

$$\langle n_{\{ij\}}(t)\rangle^{\{(H)\}} = n\frac{a^2}{8}\frac{Q}{\left(P_x cos^2(\vartheta) + P_y sin^2(\vartheta)\right)} sech^2\left[\frac{a^2}{4}\frac{Q}{\left(P_x cos^2(\vartheta) + P_y sin^2(\vartheta)\right)}\, z_{\{ij\}}\right]. \quad \ldots\ldots\ldots\ldots\ldots\ldots (33)$$

By taking into account Eq.(31) into Eq.(16), the energy can be calculated by taking the integration in the transverse direction zij as $En\ =\ \langle \Psi n(t)|H|\Psi n(t)\rangle^{H}$ to give:

$$E_n = n\frac{a^4}{48}\frac{Q^2}{\left(P_x cos^2(\vartheta) + P_y sin^2(\vartheta)\right)^2}\left\{v_{\{g1\}}\, cos(\vartheta) + v_{\{g2\}sin(\vartheta)} + \frac{2(n-1)a}{5}\left[2J_2 - \sqrt{J_1^2 + D^2}\left(cos\big(a(k_0 + k_1 + \eta_1)\big) + cos\big(a(k_0 + k_2 + \eta_2)\big)\right)\right]\right\}. \ldots\ldots\ldots\ldots\ldots\ldots (34)$$

It is obvious that the energy is a function of n and then is quantized (see Fig.3). This energy is plotted in Fig.(3). Another form of solution is the dark solitary wave given by

$$\Psi(x, y, \tau) = \Psi_0 tanh(\mu_1 x + \mu_2 y - v_e\tau) exp\left(I(\eta_1 x + \eta_2 y - v_p\tau)\right) \ldots\ldots\ldots\ldots\ldots\ldots (35)$$

With

$$v_e\ = 2P_y\mu_2\eta_2 + 2P_x\mu_1\eta_1, v_p\ = P_x\ (\eta_1^2 + 2\mu_1^2) + P_y(\eta_2^2 + 2\mu_2^2) \ldots\ldots\ldots\ldots\ldots\ldots (36)$$

with the constraints $P_x\mu_1^2 + P_y\mu_2^2\ = -\frac{Q}{2}\Psi_0^2$. By setting again $\mu_1\ = \mu cos(\vartheta), \mu_2\ = \mu sin(\vartheta)$, it is obvious that

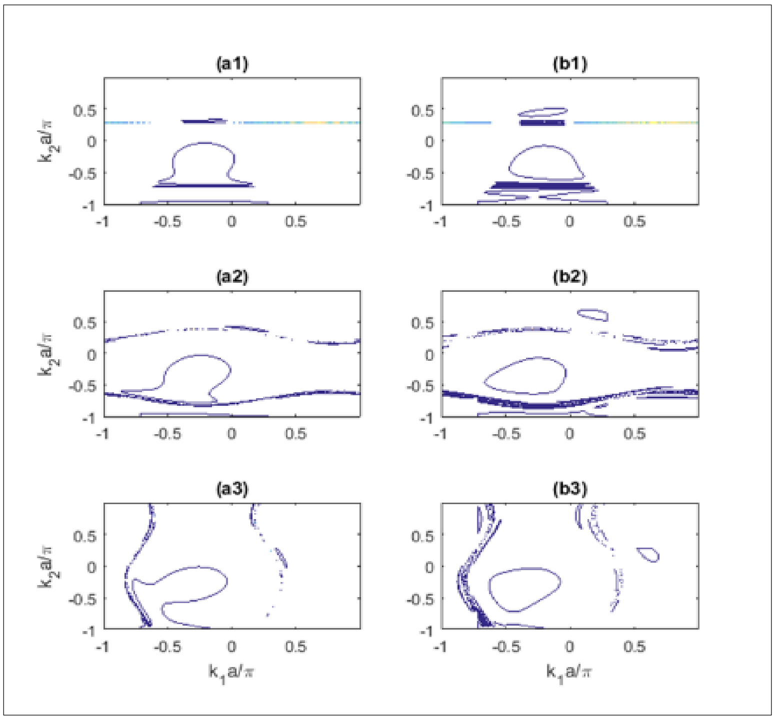

Figure 3 Contour plot of the energy given by Eq.(34), with the same parameters as in Fig.(1) and with $D/J_1\ =\ 0.8$, with $\mu_2\ =\ 0, \eta_1\ =\ \mu_1, \eta_2\ =\ \mu_2$ (a) n = 5, (b): n = 15. (1): $\vartheta =\ 0$, (2): $\vartheta = \pi/6$, (3): $\vartheta = \pi/3$

$$\mu = \Psi_0\sqrt{\frac{-Q}{2\left(P_x cos^2(\vartheta)+P_y sin^2(\vartheta)\right)}}. \ldots\ldots\ldots\ldots\ldots\ldots (37)$$

The profile of this solution is given by Fig.4. In Fig.(5), one has in blue color the domain where PxQ < 0 and PyQ < 0, and then 2-D dark soliton exists ∀ϑ. Let us find Ψ(x, y, τ) with linear phase in polar coordinate, $\Psi(x,y,\tau) = f(x,y,\tau)\, exp[I(\eta_1 x + \eta_2 y - v_p\tau)]$, leading by equating the real and imaginary parts of Eq.(23) to the following set of ordinary differential equations:

$$\frac{\partial f}{\partial \tau} + 2\left[\eta_1 P_x + \left(-\gamma_{\{1x\}}\eta_1 + 3\gamma_2(\eta_1+\eta_2) + 3\chi_1 - \chi_x\right)f^2\right]\frac{\partial f}{\partial x} + 2\left[P_y\eta_2 + \left(-\gamma_{\{1y\}}\eta_2 + 3\gamma_2(\eta_1+\eta_2) + 3\chi_1 - \chi_y\right)f^2\right]\frac{\partial f}{\partial y} = 0 \ldots\ldots\ldots\ldots\ldots\ldots (38)$$

$$f\left(v_p - P_x\eta_1^2 - P_y\eta_2^2\right) + \left(P_x + \gamma_{\{1x\}}f^2\right)\frac{\partial^2 f}{\partial x^2} + \left(P_y + \gamma_{\{1y\}}f^2\right)\frac{\partial^2 f}{\partial y^2} + \left(Q - \gamma_{\{1x\}}\eta_1^2 - \gamma_{\{1y\}}\eta_2^2 - \gamma_2(\eta_1+\eta_2)^2 - \chi_1(\eta_1+\eta_2) - \left(\eta_1\chi_x + \eta_2\chi_y\right)\right)f^3 + 3\gamma_2 f^2\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)^2 f + 6\gamma_2 f\left(\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\right)^2 + \gamma_3 f\left(\frac{\partial^2 f^2}{\partial x^2} + \frac{\partial^2 f^2}{\partial y^2}\right) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (39)$$

Equation (39) is rich and can inspire researchers to analyze and find the solutions of this class of partial differential equation.

4.2. Exotic solitons as solution: Bright compacton and peakon-like quantum soliton

Looking for traveling wave solutions in the form *f*(*x, y, t*) = *f*(*z*) with *z* =*x* cos(ϑ) + *y* sin(ϑ) - *veτ*, where *ve* is the envelope velocity, the above system is transformed into the following system of nonlinear ordinary differential equation:

$$\{-v_e + 2\left(\eta_1 P_x cos(\vartheta) + \eta_2 P_y sin(\vartheta)\right) + 2\left[\left(-\gamma_{\{1x\}}\eta_1 + 3\gamma_2(\eta_1+\eta_2) + 3\chi_1 - \chi_x\right)cos(\vartheta)\left(-\gamma_{\{1y\}}\eta_2 + 3\gamma_2(\eta_1+\eta_2) + 3\chi_1 - \chi_y\right)sin(\vartheta)\right]f^2\}f' = 0 \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

$$[P_x cos^2(\vartheta) + P_y sin^2(\vartheta) + f^2(\gamma_{\{1x\}}cos^2(\vartheta) + \gamma_{\{1y\}}sin^2(\vartheta) + 3\gamma_2(1+sin(2\vartheta)) + 2\gamma_3)]f'' + \left(Q - \gamma_{\{1x\}}\eta_1^2 - \gamma_{\{1y\}}\eta_2^2 - \gamma_2(\eta_1+\eta_2)^2 - \chi_1(\eta_1+\eta_2) - \left(\eta_1\chi_x + \eta_2\chi_y\right)\right)f^3 + 2\left[3\gamma_2\left(1+sin(2\vartheta)\right) + \gamma_3\right]ff'^2 + f\left(v_p - P_x\eta_1^2 - P_y\eta_2^2\right) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (41)$$

where the prime indicates the derivatives with respect to *z*. From Eq.(40), it is obvious that

$$v_e = 2(\eta_1 P_x cos(\vartheta) + \eta_2 P_y sin(\vartheta)), \qquad \ldots\ldots\ldots\ldots\ldots\ldots (42)$$

and

$$\left(3\gamma_2(\eta_1+\eta_2) + 3\chi_1 - \chi_x - \gamma_{\{1x\}}\eta_1\right)cos(\vartheta) + \left(3\gamma_2(\eta_1+\eta_2) + 3\chi_1 - \chi_y - \gamma_{\{1y\}}\eta_2\right)sin(\vartheta) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (43)$$

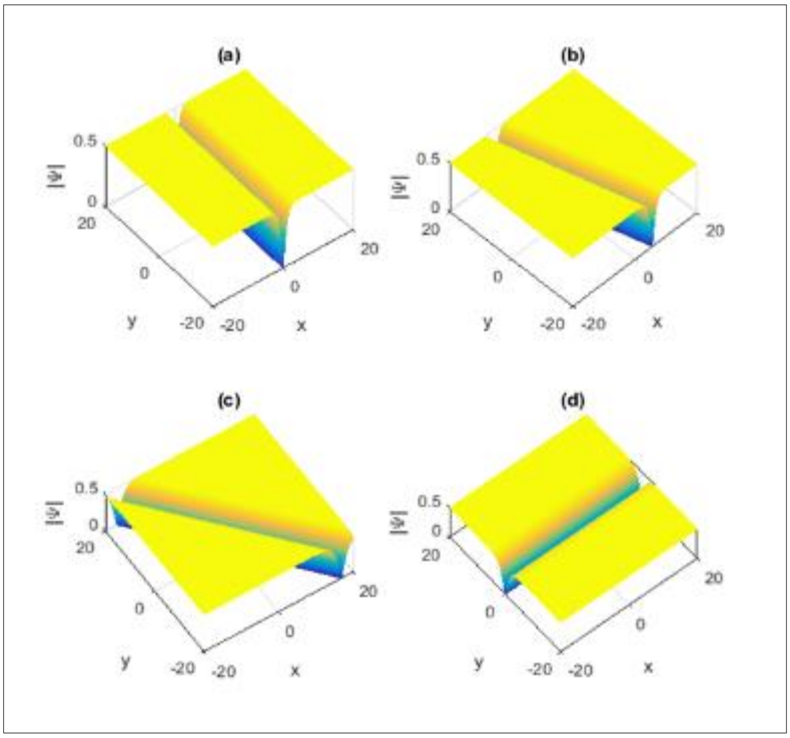

Figure 4 Profile of 2D Dark soliton given by Eq.(35). with parameters $P_x = 0.5$, $P_y = 0.2$, $Q = -1$ and $\Psi 0 = 0.5$, (a): $\mu_1 = 0$, (b): $\mu_1 = 0.2$. (c) $\mu_1 = 0.4$, (d) $\mu_1 = 0.5$, while μ_2 is deduced from Eq.(37)

4.2.1. Pulse compacton-like quantum signal

Let us mention that, pulse compacton is the solution of Eq.(52) whether $f \equiv 0$ is unconditionally the solution of Eq.(52), leading to the constraints$P_x cos^2(\vartheta) + P_y sin^2(\vartheta) = 0$, leading then to a compact bright soliton as a solution in the form [6]:

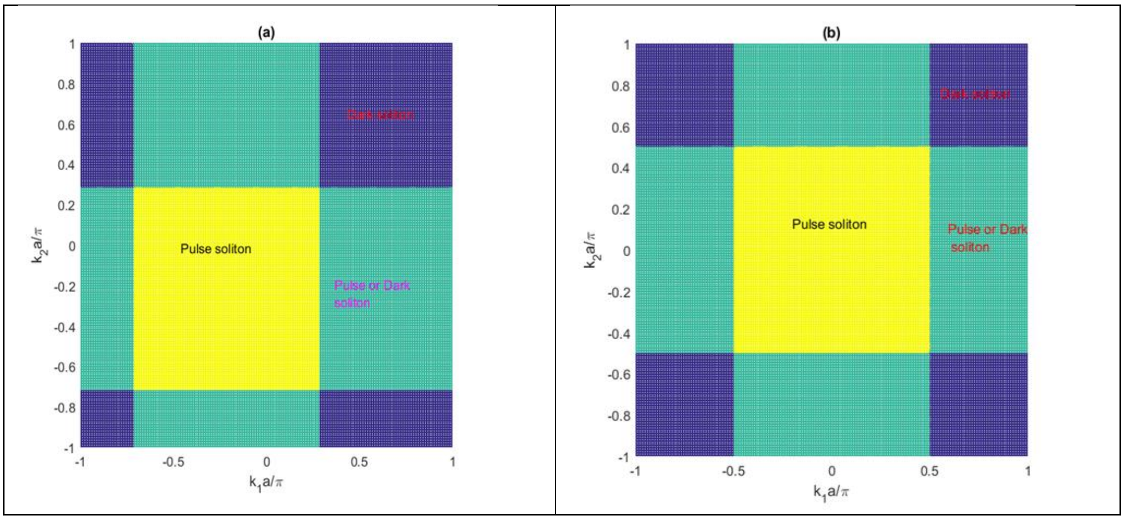

Figure 5 Domain of existence of Pulse and dark solitudes according to Eqs.(28,37). In all cases, we fix $J_1 = 0.4$ and $J_2 = 0.45$, $n = 20$ and $S = 15$. (a) $D/J_1 = 0.8$, (b) $D/J_1 = 0.0$.

$$f(z) = \begin{cases} A_0 cos\big(\mu(z - z_0)\big), if \ |z - z_0| \leq \frac{\pi}{2\gamma}, \\ 0, else \end{cases} \quad \dots\dots\dots\dots\dots(44)$$

With

$$\mu^2 = \frac{\left(Q-\gamma_{\{1x\}}\eta_1^2-\gamma_{\{1y\}}\eta_2^2-\gamma_2(\eta_1+\eta_2)^2-\chi_1(\eta_1+\eta_2)-(\eta_1\chi_x+\eta_2\chi_y)\right)}{\gamma_{\{1x\}}cos^2(\vartheta)+\gamma_{\{1y\}}sin^2(\vartheta)+9\gamma_2\left(1+sin(2\vartheta)\right)+4\gamma_3} \dots\dots\dots\dots\dots\dots(45)$$

With

$$v_p = \mu^2\left[P_x\, cos^2(\vartheta) + P_y sin^2(\vartheta) + A_0^2\left(\frac{3}{4}\gamma_2\left(1+sin(2\vartheta)\right)+\gamma_3+\frac{3}{4}\left(\gamma_{\{1x\}}cos^2(\vartheta)+sin^2(\vartheta)\gamma_{\{1y\}}\right)\right)\right]$$

$$+\frac{3}{4}A_0^2\left[\left(\gamma_{\{1x\}}+\gamma_2\right)\eta_1^2+\left(\gamma_{\{1y\}}+\gamma_2\right)\eta_2^2-Q+\eta_1\eta_2\gamma_2+\eta_1(\chi_1+\chi_x)+\frac{3}{4}\eta_2\left(\chi_1+\chi_y\right)\right]+P_x\,\eta_1^2+P_y\eta_2^2. \ (46)$$

Compact solution (44) satisfies the normalization condition

$$\rho = \sum_{\{i=1\}}^{\{f_1\}}\sum_{\{j=1\}}^{\{f_2\}}\left|\Phi_{\{n,i,j\}}(t)\right|^2 = \frac{A_0^2}{a}\int_{\left\{z_0-\frac{\pi}{2\mu}\right\}}^{\left\{z_0+\frac{\pi}{2\mu}\right\}} cos^2(\mu\,(z-z_0))dz = \frac{\pi\,A_0^2}{2\gamma\,a} = 1 \dots\dots\dots\dots\dots\dots(47)$$

leading that $A0 = \sqrt{2\mu a/\pi}$. Remembering to original variable, it is obvious that:

$$\Psi(x,y,\tau) = \begin{cases} \sqrt{\frac{2\mu\,a}{\pi}}cos\left(\mu(xcos(\vartheta)+ysin(\vartheta)-v_e\tau)\right)exp\left[I\left(\eta_1 x+\eta_2 y-v_p\tau\right)\right], \\ \quad if\ \mu|xcos(\vartheta)+ysin(\vartheta)-v_e\tau| \le \frac{\pi}{2\mu}. \\ \quad 0, else \end{cases} \dots\dots\dots\dots\dots\dots (48)$$

The profile of this solution is shown in Fig(6). One can here construct the following Hartree product eigenstates:

$$|\Psi_n(t)\rangle^{\{(H)\}} = \frac{1}{\sqrt{n!}}exp\left(-nI\left(\omega+\eta_1\,v_{\{g1\}}+\eta_2 v_{\{g2\}}+v_p\right)t\right)\left(\frac{2\mu\,a}{\pi}\right)^{\frac{n}{2}}\left(\sum_{\{i=1\}}^{f}\sum_{\{j=1\}}^{f}\left(\cos\left[a\mu\left(icos(\vartheta)+jsin(\vartheta)\right)\right]\right]exp\ I[(k_1+\eta_1)ia+(k_2+\eta_2)ja]a_{\{ij\}}^{\dagger}\right)^n|0\rangle. if\ |icos(\vartheta)+jsin(\vartheta)| \le \frac{\pi}{2a\mu}. \dots\dots\dots\dots\dots\dots (49)$$

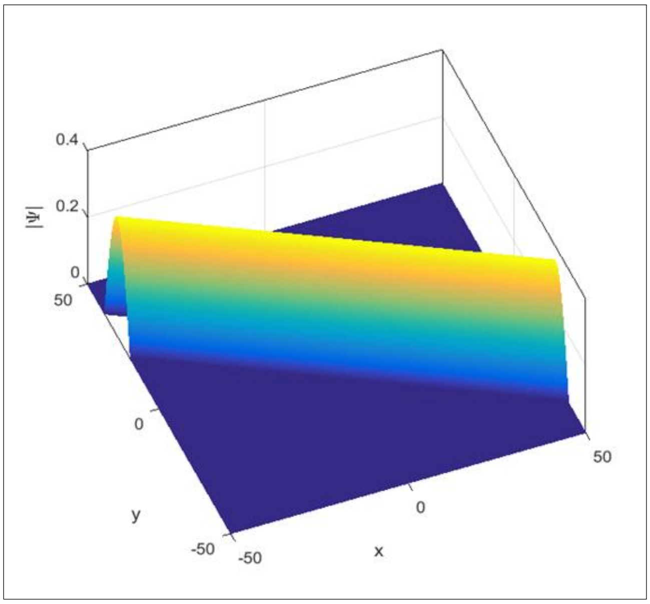

Figure 6 Profile of 2D Pulse compacton given by Eq.(48). with parameters $\mu = 0.2$, $a_1 = \frac{\sqrt{3}}{2}$, $a_2 = 1/2$ and $a = 1$.

4.3. 2D- Peakon-like quantum signal

Next, assuming a peak soliton as a solution in the form :

$$f(z) = \beta exp(-\mu\,|z - z_0|), \ldots\ldots\ldots\ldots\ldots\ldots (50)$$

one has the phase velocity:

$$v_p = P_x\eta_1^2 + P_y\eta_2^2 - \mu^2\left(P_x\,cos^2(\vartheta) + P_y\,sin^2(\vartheta)\right) \ldots\ldots\ldots\ldots\ldots\ldots (51)$$

and the widths

$$\mu^2 = -\frac{\left(Q-\gamma_{\{1x\}}\eta_1^2-\gamma_{\{1y\}}\eta_2^2-\gamma_2(\eta_1+\eta_2)^2-\chi_1(\eta_1+\eta_2)-(\eta_1\chi_x+\eta_2\chi_y)\right)}{\gamma_{\{1x\}}cos^2(\vartheta)+\gamma_{\{1y\}}sin^2(\vartheta)+9\gamma_2\left(1+sin(2\vartheta)\right)+4\gamma_3}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (52)$$

The above solution must satisfy the normalization condition (12), one has

$$\rho = \sum_{\{i=1\}}^{f_1}\sum_{\{j=1\}}^{f_2}\left|\Phi_{\{n,ij\}}(t)\right|^2 = \rho = \sum_{\{i=1\}}^{\{f_1\}}\sum_{\{j=1\}}^{\{f_2\}}\left|\Phi_{\{n,i,j\}}(t)\right|^2 = \frac{\beta^2}{a}\int_{\{-\infty\}}^{\{+\infty\}} exp\,(-2\mu|\eta|)dz = \frac{\beta^2}{\mu a} = 1, \ldots\ldots\ldots\ldots\ldots\ldots \quad (53)$$

leading to the peakon amplitude $\beta = \sqrt{\mu a}$.

5. Conclusion

In this paper we have studied the 2-D compacton and 2-D peakon-like quantum states in a two-dimensional ferromagnet XXZ spin chain with Dzyaloshinsky-Moriya interaction. Based on the time-dependent Hartree approximation, we have shown that the quantum states may be governed by the 2-D discrete nonlinear Schrödinger (DNLS) equation. Next, using the semi-discrete multiple-scale method, we have shown that the 2-D DNLS can be reduced to the 2-D continuum extended nonlinear Schrödinger (ENLS) equation which consists of the basic NLS equation with additional nonlinear dispersive terms, admitting two types of exact solitary wave as solutions: The 2-D bright compacton and 2-D peakon as eigenstates, according to the relative magnitude of its coefficients. Next we have found the energy levels formula of these quantum pulse soliton is quantized. We found also that on the contrary to results found for the classical cases where initial amplitudes of both solutions are free parameters, the initial amplitudes are not free parameters since the obtained solutions need to be normalized.

Compliance with ethical standards

Acknowledgments

We would like to express special thanks to Pr Z. I. Djoufack for its expertise throughout all aspects of the present work

Disclosure of conflict of interest

All of the authors declare that they have all participated in the design, execution, and analysis of the paper, and that they have approved the final version. Additionally, there are no conflicts of interest in connection with this paper, and the material described is not under publication or consideration for publication elsewhere.

References

[1] Kenmogne F, Yemélé D, Kengne J and Ndjanfang D 2012 Phys. Rev. E 90052921

[2] Rosenau P and Hyman J M 1993 Phys. Rev. Lett. 70 564

[3] Marquie P, Bilbault J M, and Remoissenet M 1994 Phys. Rev. E 49 828

[4] Marquie P, Bilbault J M, and Remoissenet M 1995 Phys. Rev. E 51 6127

[5] Kenmogne F and YeméléD 2012 Chaos, Solitons, Fractals 45 21-34

[6] Donkeng H-Y, Kenmogne F, Yemélé D, Jeutho M G, Mabou W K and NDjanfang D 2017 Chinese Journal of Physics 55 683691

[7] Kenmogne F, YeméléD and Woafo P 2012 Phys. Rev. E 85 056606

[8] Pokam N C, YeméléD, Donkeng H-Y and KofanéT C 2017 Communication in nonlinear Science and numerical simulation 43 50-61

[9] Pokam N C, Fewo S I and YeméléD 2017 Phys. Rev. E 95 012215

[10] Sievers A J and Takeno S 1988 Phys. Rev. Lett. 61 970

[11] Campbell D K, Flach S and Kivshar Y S 2004 Phys. Today 57 43

[12] Djoufack Z I, Kenfack-Jiotsa A, Nguenang J P and Domngang S 2010 J. Phys. Condens. Matter 22 205502

[13] Djoufack Z I, Kenfack-Jiotsa A and Nguenang J P 2012 Eur. Phys. J. B 85 96

[14] Tang B, Li D-J, and Tang Y 2014 Chaos 24 023113

[15] Djoufack Z I, Tala-Tebue E, Nguenang J P and Kenfack-Jiotsa A 2016 Chaos 26 103110

[16] Fabien Kenmogne, Zacharie I Djoufack, Emmanuel Foadieng, Didier Fokwa, David Yemélé, Phys. Scr. 95 (2020) 045108 (13pp)

[17] Dzyaloshinsky I 1958 J. Phys Chem. Solids 4 241

[18] Moriya T 1960 Phys. Rev. Lett. 4 228

[19] Sznajd J 2003 Phys. Rev. B 68 174420

[20] Dyson F J 1956 Phys. Rev. 102 1217; 102 1230

[21] Wright E, Eilbeck J C, Hays M H, Miller P D and Scott A C 1993 Physica D 69 18

[22] Kenmogne F 2015 J Appl Computat Math 4:1

[23] Li D J and Tang Y 2012 Mod. Phys. Lett. B 26 1250160

[24] Lai Y and Haus H A 1989 Phys. Rev. A 40 844

[25] Wright E M 1991 Phys. Rev. A 43 3836

[26] Remoissenet M 1986 Phys. Rev. B 33 2386